Week 7

Numerical methods for physicists, 2018/19 autumn semester

Eigenvalue problems - `eig, eigs, hess`

PROBLEM 1. By the use of Gershgorin's theorem give estimations for the eigenvalues of the matrix

$$\mathbf{A} = \begin{bmatrix} 4.2 & -0.1 & 0.2 & 0.1 \\ 0.05 & 3 & -0.1 & -0.05 \\ 0.5 & -0.5 & 2 & 0.1 \\ 0.1 & 0.2 & 0.3 & -1 \end{bmatrix}.$$

Give an estimation for the spectral radius of the matrix.

PROBLEM 2. By the use of the Bauer–Fike theorem estimate the change of the eigenvalues of the matrices in the case when we add 0.1 to the element $a_{2,1}$.

$$a)\ \mathbf{A} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}, \quad b)\ \mathbf{A} = \begin{bmatrix} 9 & 2 \\ 1 & 3 \end{bmatrix}$$

PROBLEM 3. Compute the strictly (or single) dominant eigenvalue and the eigenvector of the matrix

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

using the power method with the starting vector $\overline{\mathbf{x}}_0 = [6, 7, 3]^T$. Compute the eigenvalue (and the eigenvector) closest to 1 starting from the vector $[3, -3, -5]^T$. Solve the problems manually (two iteration steps are enough) and by the use of the provided Matlab code. What is the second largest eigenvalue in absolute value?

PROBLEM 4. Let the matrix $\mathbf{A}$ be defined by the Matlab command `toeplitz([20:-1:1])`. Approximate all the eigenvalues of the matrix with the QR iteration. Give the error of the approximations after 100 steps.

PROBLEM 5. Compute the eigenvalues of the matrix

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

manually using the QR iteration. How can we make the algorithm usable?

PROBLEM 6. Reduce (manually) matrix $\mathbf{A}$ to an upper Hessenberg form such that its eigenvalues do not change. Apply the QR iteration with the new reduced matrix.

$$\mathbf{A} = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 1 & 2 \\ 4 & 2 & 1 \end{bmatrix}$$

HOMEWORK FOR WEEK 7 - to be submitted until the next computer lab (The detailed solutions can be submitted either on A4 sheets of paper (printed or written) or in a pdf file (e.g. in an exported Matlab livescript) to rhorvath@math.bme.hu. Do not send Matlab files. Answer all questions with a sentence at the end of each problem.)

1. (2p) Take two steps manually with the QR iteration for the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix},$$

then give estimations for the eigenvalues of $\mathbf{A}$.

2. (2p) The matrix $\mathbf{A}$ is defined in Matlab as `A=toeplitz([2,-1,ones(1,18)])`. Give the following eigenvalues of the matrix by the use of the power method (and its variants) in Matlab. Give also the codes that produce the desired result.

a) the largest eigenvalue in absolute value,

b) the closest eigenvalue to 1,

c) the smallest eigenvalue in absolute value.