Week 5

Numerical methods for physicists, 2018/19 autumn semester

Partial pivoting, general LU decomposition - `lu`

Problem. 1. Solve the system

$$\begin{aligned} 10^{-5}x + y &= 1 \\ x + y &= 2 \end{aligned}$$

with the Gaussian method without and with partial pivoting using decimal floating point numbers with four-digit-long mantissas. Compare the results. Give the general LU decomposition of the coefficient matrix in exact arithmetic.

Cholesky decomposition - `chol`

Problem. 2. The LU decomposition of the matrix

$$\mathbf{A} = \begin{bmatrix} 4 & 2 & 1 \\ 2 & 17 & 13/2 \\ 1 & 13/2 & 23/2 \end{bmatrix}$$

is given in concise form

$$\begin{bmatrix} 4 & 2 & 1 \\ 1/2 & 16 & 6 \\ 1/4 & 3/8 & 9 \end{bmatrix}.$$

Give the Cholesky decomposition of $\mathbf{A}$ if it exists.

Problem. 3. Give the Cholesky decomposition of the matrix $\mathbf{A}$ in Problem 2 directly, that is without computing the LU decomposition.

Problem. 4. Solve the system $\mathbf{A}\overline{\mathbf{x}} = [3, -9/2, -21/2]^T$ ($\mathbf{A}$ is the matrix from Problem 2) using the Cholesky decomposition of the matrix.

Iterative solutions of linear systems (Jacobi and Gauss-Seidel iterations and their relaxed versions)

Problem. 5. Solve the system

$$\begin{bmatrix} -1 & 5 & -2 \\ 1 & 1 & -4 \\ 4 & -1 & 2 \end{bmatrix} \overline{\mathbf{x}} = \begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix}$$

with an appropriate (!) iterative solver. Write the computer code in Matlab. Estimate the number of the necessary iteration steps if we would like to achieve an error of 10^{-9} in maximum norm and the iteration starts at $\overline{\mathbf{x}}_0 = [0, 0, 0]^T$.

Problem. 6. Which iterative solvers can be used to solve the system

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \overline{\mathbf{x}} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}?$$

Compute the optimal choice for the relaxation parameter ω in the JOR method. Compare the number of iterations needed to achieve an error of 10^{-6} in maximum norm with the optimal ω parameter and with $\omega = 0.01$ (we start the iteration from the zero vector).

HOMEWORK FOR WEEK 5 - to be submitted until the next computer lab (The detailed solutions can be submitted either on A4 sheets of paper (printed or written) or in a pdf file (e.g. in an exported Matlab livescript) to rhorvath@math.bme.hu. Do not send Matlab files. Answer all questions with a sentence at the end of each problem.)

1. (2p) Calculate the solution of the system

$$\begin{aligned} 0.003x_1 + 59.14x_2 &= 59.17 \\ 5.291x_1 - 6.13x_2 &= 46.78 \end{aligned}$$

manually using the Gaussian method with and without partial pivoting such that we round all computed values to four-digit-long mantissas. Compare the result with the exact solution $x_1 = 10, x_2 = 1$.