Week 11

Numerical methods for physicists, 2018/19 autumn semester

Trigonometric interpolation - `fft, ifft`

PROBLEM 1. Let us construct the trigonometric interpolation polynomial to the function $f(x) = |x - \pi|$ on 4 equidistant interpolation nodes.

PROBLEM 2. The values of a function f at 6 equidistant nodes on the interval $[0, 2\pi]$ are $0, 1, 1, 0, -1, -1$, respectively. a) Calculate the coefficient of $\sin x$ in the trigonometric interpolation polynomial manually. b) Use Matlab's `fft` command to calculate the complex and real discrete Fourier coefficients. c) Apply the fast Fourier transform manually to calculate the complex and real discrete Fourier coefficients.

PROBLEM 3. The vector f in the code found in the m-file represents a sampling vector from a noisy 2π periodic signal. Use the fast Fourier transform to filter out the noise from the signal.

Numerical differentiation

PROBLEM 4. Calculate the approximate first derivative of the function $f(x) = 1/x$ at the point $x_0 = 0.05$ using the forward difference formula with mesh sizes $h_1 = 0.0016$ and $h_2 = 0.0008$. Use Richardson extrapolation to give a better estimate for the derivative.

PROBLEM 5. Show that the forward finite difference formula

$$D = \frac{-3f_0 + 4f_1 - f_2}{2h}$$

approximates the first order derivative with convergence order 2. Check the result on the function of the previous problem. Use Richardson extrapolation to give a better estimate for the derivative.

HOMEWORK FOR WEEK 11 - to be submitted until the next computer lab (The detailed solutions can be submitted either on A4 sheets of paper (printed or written) or in a pdf file (e.g. in an exported Matlab livescript) to rhorvath@math.bme.hu. Do not send Matlab files. Answer all questions with a sentence at the end of each problem.)

1. (2p) Let us fit an interpolation polynomial to the points $(x_{-1}, f_{-1}), (x_0, f_0), (x_1, f_1)$ $(x_0 - x_{-1} = x_1 - x_0 = h)$ and compute the first and second derivatives of the interpolation polynomial at $x = x_0$. How do these values relate to some finite difference approximations of the derivatives?

2. (2p) Let us consider the 2π periodic function

$$f(x) = \begin{cases} x, & \text{if } 0 \le x < \pi, \\ 0, & \text{if } \pi \le x < 2\pi. \end{cases}$$

Use trigonometric interpolation to interpolate the function at 60 nodes, and plot this interpolation polynomial (use Matlab `fft` function) . Compute the coefficient of $\cos x$ directly.