Week 10

Numerical methods for physicists, 2018/19 autumn semester

<u>Interpolation with polynomials - `polyfit, polyval, spline, ppval`</u>

PROBLEM 1. Construct the interpolation polynomial to the points (-1,6), (0,3), (1,2) with Lagrange's and Newton's methods.

PROBLEM 2. We interpolate the function $f(x) = \ln(x+1)$ on the nodes 0, 0.6, 0.9. Give an upper bound for the interpolation error at the point $x = 0.45$.

PROBLEM 3. We would like to interpolate the function $f(x) = \sin x$ with a piecewise linear function on the interval $[0, \pi]$ using equidistant nodes. Give an upper estimate to the step size that guarantees an interpolation error less than 0.001 on the whole interval.

PROBLEM 4. We interpolate Runge's function $f(x) = 1/(1+x^2)$ in the interval $[-1, 1]$ on 12 Chebyshev nodes. Estimate the interpolation error (use Matlab to compute and estimate the derivatives of the function).

PROBLEM 5. Give the polynomial p with the smallest degree possible such that $p(1) = 2$, $p(3) = 1$, $p'(1) = 1$ and $p'(3) = 2$.

PROBLEM 6. Construct the piecewise cubic natural spline that interpolates the points (1,2), (2,1), (3,1).

HOMEWORK FOR WEEK 10 - to be submitted until the next computer lab (The detailed solutions can be submitted either on A4 sheets of paper (printed or written) or in a pdf file (e.g. in an exported Matlab livescript) to rhorvath@math.bme.hu. Do not send Matlab files. Answer all questions with a sentence at the end of each problem.)

1. (2p) Let us interpolate the function $f(x) = \sqrt[4]{x} + x - 2$ on the following three nodes: $x = 16, 625/16, 81$. Give the interpolation polynomial (simplification is not necessary). Give an upper bound for the error of the interpolation at the point $x = 25$.
2. (2p) We interpolate the function $f(x) = x^3$ on three Chebyshev nodes in the interval [-1,1]. Construct the interpolation polynomial and show manually that the interpolation error is not greater than 1/4!
3. (2p) Construct the piecewise cubic clamped spline s that interpolates the points (1,2), (2,1), (3,1) and satisfies the conditions $s'(1) = 2$ and $s'(3) = -1$. Give the system of equations to be solved, solve it with Matlab and give the polynomials on each separate interval.