Homework for week 4

Appl. numer. meth. with Matlab, 2019/20 spring semester

Homework is for independent work at home. It must be submitted until the next computer lab. The detailed solutions can be submitted either on an A4 sheet of paper (printed or written) or in a pdf file (e.g. in an exported Matlab livescript) to rhorvath@math.bme.hu. Do not send Matlab files. Give explicit answers to the questions of the problems and evaluate the results.

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1. (2p) (Matlab) Show by giving an estimation for the spectral radius of the iteration matrix that the SOR method with $\omega = 1/2$ can be used to solve system

$$\begin{aligned} 9x_1 - 3x_2 &= 6 \\ -2x_1 + 8x_2 &= -4. \end{aligned}$$

Estimate the number of iteration steps to achieve an approximation of the exact solution with the error 10^{-8} (in 1-norm and starting from the zero vector). Perform the iteration in Matlab and give the corresponding approximation.

2. (2p) (Matlab) Let us consider the linear system $Ax = b$, where $A = 20I - bucky$, where I is the 60×60 identity matrix, *bucky* is one of Matlab's built-in matrices and b is a column vector with elements $1, 2, \ldots, 60$. (In order to decrease the CPU time, let us define the identity matrix as a sparse matrix.) Let us visualize the nonzero structure of the coefficient matrix. Let us show that the system can be solved with the Jacobi relaxation method with relaxation parameter $\omega = 1/2$. (Hint: show that the matrix is an M-matrix.) Construct the iteration and iterate starting from the zero vector until the maximum norm of the residual vector is less than 10^{-6}. How many steps does the algorithm need to achieve this accuracy?