Homework for week 3

Appl. numer. meth. with Matlab, 2019/20 spring semester

Homework is for independent work at home. It must be submitted until the next computer lab. The detailed solutions can be submitted either on an A4 sheet of paper (printed or written) or in a pdf file (e.g. in an exported Matlab livescript) to rhorvath@math.bme.hu. Do not send Matlab files. Give explicit answers to the questions of the problems and evaluate the results.

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1. (2p) (Manually) Let us consider the system of linear algebraic equations

$$\mathbf{A}\overline{\mathbf{x}} = \begin{bmatrix} 1 & 2 & -4 & -15 \\ 2 & -1 & 1 & 0 \\ 1 & -5 & 4 & -2 \\ 2 & -2 & 2 & 1 \end{bmatrix} \overline{\mathbf{x}} = \begin{bmatrix} 1 \\ 3 \\ -2 \\ 4 \end{bmatrix} = \overline{\mathbf{b}},$$

where the inverse of $\mathbf{A}$ has the form

$$\mathbf{A}^{-1} = \begin{bmatrix} 1 & -10 & -2 & 11 \\ -11 & 118 & 21 & -123 \\ -13 & 139 & 25 & -145 \\ 2 & -22 & -4 & 23 \end{bmatrix}.$$

Let us give an upper estimation in 1-norm for the relative change of the solution of the system caused by the following change of the coefficients of the equations: we add random numbers δa_{ij} to the elements a_{ij}, respectively, with the property $|\delta a_{ij}| \leq 0.0005$ and we modify the elements of $\overline{\mathbf{b}}$ at most with the 3% of the element.

2. (2p) (Manually) Give the LU and Cholesky decompositions of the matrix

$$\begin{bmatrix} 4 & 1 & -2 \\ 1 & 17/4 & 1/6 \\ -2 & 1/6 & 19/9 \end{bmatrix}.$$

3. (2p) (Matlab) In Matlab, we generally solve linear systems given in the form $Ax = b$ with the left division command: $x = A\backslash b$. Using this command appropriately, perform the computations as follows.

Let B be a random 5000×5000 real matrix (`rand(5000)`) and define $A = B^T B + (5000^2/4)\ I$, where I is the identity matrix. Moreover, let b a 5000 element random column vector. Solve the system $Ax = b$ in Matlab in three different ways: using the LU decomposition (`lu`), using the Cholesky decomposition (`chol`) and using Matlab's built-in solver (`A\b`). Measure the CPU time of the methods with the `tic-toc` commands. Compare and explain the obtained results.