

Homework for week 2

Appl. numer. meth. with Matlab, 2019/20 spring semester

Homework is for independent work at home. It must be submitted until the next computer lab. The detailed solutions can be submitted either on an A4 sheet of paper (printed or written) or in a pdf file (e.g. in an exported Matlab livescript) to rhorvath@math.bme.hu. Do not send Matlab files. Give explicit answers to the questions of the problems and evaluate the results.

1. (2p) (Manually + Matlab) Let us consider vector v and matrix A as follows

$$v = \begin{bmatrix} -3 \\ 4 \\ -5 \\ 7 \end{bmatrix}, \quad A = \begin{bmatrix} -1 & 2 & 3 \\ 4 & -3 & 5 \\ 2 & 2 & 0 \end{bmatrix}.$$

Give the 1-, 2-, ∞ - and Frobenius norms of them manually (in the case of the 2-norm of A , we may use Matlab). Is it true that $\rho(A) < 7.2$? What is the condition number of A in maximum norm (Matlab)?

2. (2p) (Matlab) Let us apply the difference quotient

$$d_i = \frac{f(x_0 + h_i) - f(x_0)}{h_i}$$

to approximate the derivative of the function $f(x) = \sin^2(x)$ at the point $x_0 = \pi/4$ (exact value is 1). Let us use the values $h_i = 0.1/2^i$ ($i = 1, \dots, 15$). Compute the errors of the approximation for all h_i values, and use the graphical method to estimate the order of the convergence of the sequence d_i .

3. (2p) (Manually) Let us consider the quadratic equation $x^2 - 1000.001x + 1 = 0$ and "solve" it on a computer that uses decimal numbers with 5-digit-long mantissas (there is no restriction to the characteristic) using the well-known solution formula for quadratic equations (we have to perform a rounding after each operation). How could we get better result on the same computer? What is the machine precision of this machine?