1° a) $\dot r = (3, 6t, 6t^2)$, $|\dot r| = \sqrt{9+36t^2+36t^4} = 6t^2+3$

$|L| = \int_0^1 (6t^2+3)\,dt = [2t^3+3t]_0^1 = 5$

b) akkor konz. erőtér, ha [illegible] összeggel. Feld. konzervatív [illegible] lehet [illegible] [illegible] [illegible] konvergens, s a lehetséges [illegible]

2° $\oint_L f\,dr = \int_T (f'_{2x} - f'_{1y})\,d(x,y) = \int_0^1 \int_0^{x^3} 2xy^2\,dy\,dx = \int_0^1 2x\left[\frac{y^3}{3}\right]_0^{x^3} dx = \int_0^1 \frac{2}{3}x^{10}\,dx = \frac{2}{3\cdot 11}$

$8xy$, $6xy^2$ (x) (y)

3° a) $r(x,y) = (x, y, 4-x-y)$ $0\le x\le 1$, $0\le y\le 1$, $r'_x \times r'_y = \begin{vmatrix} i & j & k \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{vmatrix} = (1,1,1)$

$\iint_F f\,|dF| = \int_0^1\int_0^1 (4-x-y)\sqrt3\,dy\,dx = \sqrt3 \int_0^1 \underbrace{\left(4-x-\frac12\right)}_{\frac72 - x} dx = \sqrt3\left(\frac72 - \frac12\right) = 3\sqrt3$

b) $x\in\mathbb{R}^d$ torlódási pontja A-nak, ha $\forall r>0$-ra $B_x(r)\cap A$-nak végtelen sok eleme van

$x\in\mathbb{R}^d$ határpontja A-nak, ha $\forall r>0$-ra $B_x(r)\cap A \ne \emptyset$ és $B_x(r)\setminus A \ne \emptyset$

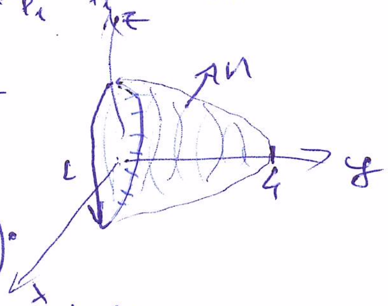
4° a) $|T| = \oint_L f\,dr$, $f(x,y) = (0, x)$

b) $g(t) = f(a+t(x-a))$, $T_r f(a,x) = f(a) + \sum_{k=1}^{r} \frac{g^{(k)}(0)}{k!}$, a maradéktag

$g^{(k)}(0) = \sum_{\ell_1=1}^{d} \dots \sum_{\ell_k=1}^{d} \frac{\partial^k f}{\partial x_{\ell_1}\dots\partial x_{\ell_k}}(a)(x_{\ell_1}-a_{\ell_1})\dots(x_{\ell_k}-a_{\ell_k})$

$\frac{g^{(r+1)}(t_0)}{(r+1)!}$ egy $0\le t_0\le 1$-re,

$g^{(r+1)}(t_0) = \sum_{\ell_1=1}^{d} \dots \sum_{\ell_{r+1}=1}^{d} \frac{\partial^{r+1} f}{\partial x_{\ell_1}\dots\partial x_{\ell_{r+1}}}(a+t_0(x-a))\cdot(x_{\ell_1}-a_{\ell_1})\dots(x_{\ell_{r+1}}-a_{\ell_{r+1}})$



5° a) Legyen $y=0$, $4x^2+z^2=4$ ellipszis, a pos. y fel- felület felől nézve $+$

$r(t) = (\sin t, 0, 2\cos t)$, $0\le t\le 2\pi$

b) $r(t) = \left(\sin t, 0, \sin t + \frac{1}{4+2\cos t}\right)$

$\int_F \operatorname{rot} f\,dF = \oint_L f\,dr = \int_0^{2\pi}\left(-2\cos t + \frac{1}{2+\sin t}, 0, \sin t + \frac{1}{4+2\cos t}\right)\cdot(\cos t, 0, -2\sin t)\,dt = \int_0^{2\pi}\left(\underbrace{-2\cos^2 t - 2\sin^2 t}_{-2} + \frac{\cos t}{2+\sin t} - \frac{2\sin t}{4+2\cos t}\right)dt$

$= -4\pi + \underbrace{\left[\ln(2+\sin t) + \ln(4+2\cos t)\right]_0^{2\pi}}_{0} = -4\pi$

6° a) $\iint_F f\,dF = \int_V \operatorname{div} f\,dV = \int_0^a\int_0^b\int_0^1 (-2x-4y-6z+12)\,dz\,dy\,dx = \int_0^a\left((-2x+9)b - 2b^2\right)dx = 9ab - 2ab^2 - a^2b$

$= \int_0^a\int_0^b (-2x-4y+12-3)\,dy\,dz = ab(9-2b-a) \ge 0$, ha $a+2b\le 9$, a Δ határán $0 \Rightarrow$ belsejében van a lok max

b) $g(a,b) = 9ab - 2ab^2 - a^2b$

$0 = g'_a = 9b - 2b^2 - 2ab \overset{b\ne 0}{\Rightarrow} 9 = 2b + 2a$

$0 = g'_b = 9a - 4ab - a^2 \overset{a\ne 0}{\Rightarrow} 9 = 4b + a$ $\Big|$ $a = 2b$, $b = \frac32$, $a = 3$

$g''_{aa} = -2b$, $g''_{ab} = 9 - 4b - 2a$, $g''_{bb} = -4a$

$g'' = \begin{pmatrix} -3 & -3 \\ -3 & -12 \end{pmatrix} \to$ neg def $\Rightarrow$ lok max