1) a) $0 \le y \le 1$, $y^2 \le x \le y^{2/3} \iff 0 \le x \le 1$, $x^{3/2} \le y \le \sqrt{x}$, $\int_0^1 \int_{y^2}^{y^{2/3}} y e^{x^2} dx dy = \int_0^1 \int_{x^{3/2}}^{\sqrt{x}} y e^{x^2} dy dx = \int_0^1 \frac{x - x^3}{2} e^{x^2} dx =$ $x^2 = t$, $2x dx = dt$

$= \int_0^1 \frac{1-t}{4} e^t dt = \frac{1}{4}\left[e^t - te^t + e^t\right]_{t=0}^1 = \frac{e-2}{4}$

b) Ha $f : [K, \infty) \to [0, \infty)$ csökkenő, akkor $\sum f(n)$ konv $\iff \int_K^\infty f$ konv. impr. int és

$\int_{n+1}^\infty f \le S - S_n \le \int_n^\infty f$, ha $n \ge K$

2) $r(t) = (\sin t, 2, \cos t)$, $0 \le t \le 2\pi$, $\oint_L f\, dr = \int_0^{2\pi} f(r(t)) \dot r(t) dt = \int_0^{2\pi} (8\sin t, 4\sin^2 t, -8\sin^2 t)(\cos t, 0, -\sin t) dt$

$= \int_0^{2\pi} (8\sin t \cos t + 8 \underbrace{\sin^3 t}_{\sin t(1-\cos^2 t)}) dt = \left[4\sin^2 t - 8\cos t + \frac{8}{3}\cos^3 t\right]_0^{2\pi} = 0$

3) a) $\operatorname{rot}(r\underline{a}) = \operatorname{grad}(r) \times \underline{a} + r \underbrace{\operatorname{rot} a}_{0} = \frac{\underline{r} \times \underline{a}}{r} = \frac{(a_3 y - a_2 z, a_1 z - a_3 x, a_2 x - a_1 y)}{r}$

$\underline{r} \times \underline{a} = \begin{vmatrix} i & j & k \\ x & y & z \\ a_1 & a_2 & a_3 \end{vmatrix} = (a_3 y - a_2 z, a_1 z - a_3 x, a_2 x - a_1 y)$

$\operatorname{rot}(r^2 \underline{a}) = \underbrace{\operatorname{grad}(r^2)}_{(2x, 2y, 2z)} \times \underline{a} = 2\, \underline{r} \times \underline{a}$

b) a a D_f belső pontja és $\exists A_1, \dots, A_d \in \mathbb{R}$, hogy $f(x) - f(a) - \sum_{i=1}^d A_i (x_i - a_i) = r(x)$ esetén $\lim_{x \to a} \frac{r(x)}{|x - a|} = 0$

4) a) $r(x,y) = (x, y, \frac{16 - y^2}{4})$, $0 \le x \le 1$, $|y| \le 4$ (mert $z \ge 0$)

$r'_x \times r'_y = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 1 & -\frac{y}{2} \end{vmatrix} = (0, \frac{y}{2}, 1)$, $|r'_x \times r'_y| = \sqrt{1 + \frac{y^2}{4}}$, csak

$\int_F f\, |dF| = \int_{0\,(x)}^1 \int_{-4\,(y)}^4 f(r(x,y)) |r'_x \times r'_y| dy dx = \int_0^1 \int_{-4}^4 x \underbrace{\sqrt{y^2+4}\sqrt{1+\frac{y^2}{4}}}_{\frac{1}{2}(y^2+4)} dy dx = \int_0^1 \frac{x}{2} dx \cdot 2 \int_0^4 (y^2+4) dy =$

$= \frac{1}{2}\left[\frac{y^3}{3} + 4y\right]_0^4 = \frac{1}{2}\left(\frac{64}{3} + 16\right)$

b) $\operatorname{sh} x = \sum_{n=0}^\infty \frac{x^{2n+1}}{(2n+1)!}$, $\cos x = \sum_{n=0}^\infty \frac{(-1)^n}{(2n)!} x^{2n}$, $x \in \mathbb{R}$

5) a) Az $x^2 + y^2 + z^2 = 3$ gömbfelület $z \ge 0$ réme (gömbi koordináták)

b) L az $x^2 + y^2 = 3$, $z = 0$ kézvonal $t \uparrow$-ban, $r(t) = (\sqrt{3}\cos t, \sqrt{3}\sin t, 0)$, $0 \le t \le 2\pi$

$\int_F \operatorname{rot} f\, dF = \oint_L f\, dr = \int_0^{2\pi} f(r(t)) \dot r(t) dt = \int_0^{2\pi} (3\sqrt{3}\sin t, 5 - 2\sqrt{3}\cos t, -2)(-\sqrt{3}\sin t, \sqrt{3}\cos t, 0) dt =$

$= \int_0^{2\pi} (-9 \underbrace{\sin^2 t}_{\int = \pi} + 5\sqrt{3} \underbrace{\cos t}_{\int = 0} - 6 \underbrace{\cos^2 t}_{\int = \pi}) dt = -15\pi$

6) a) $\operatorname{div}(f \operatorname{grad} f) = |\operatorname{grad} f|^2 + f \underbrace{\operatorname{div} \operatorname{grad} f}_{\Delta f = 0}$ miatt a Gauss-Ostrogradszkij t. miatt

$\int_F f \operatorname{grad} f\, dF = \int_V |\operatorname{grad} f|^2 dV \ge 0$

b) $|F| = \int_A \sqrt{1 + f_x'^2 + f_y'^2}\, d(x,y)$, $A = D_f \subset \mathbb{R}^2$